Introductory Physics Laboratory Methods

Penn State Erie — The Behrend College

Contents

1. Graphing

A graph is one of the most useful tools for representing, understanding, and quantifying the relationship between two variables.

- The **independent variable**—the one that you control—typically goes on the x-axis.
- The **dependent variable**—the one that responds to changes in the independent variable—goes on the y-axis.

In this case, we say that we have a graph of y as a function of x, or y vs. x.

1.1. Graphing Best Practices

In order to determine whether an apparent trend is real—whether the equation modeling the data actually means anything—we need several data points over a wide range. In this course, the rules are

- Collect *at least* six to eight data points.
- Make the largest value of the independent variable a factor of ten (or more) larger than the smallest value.

All graphs should have the following:

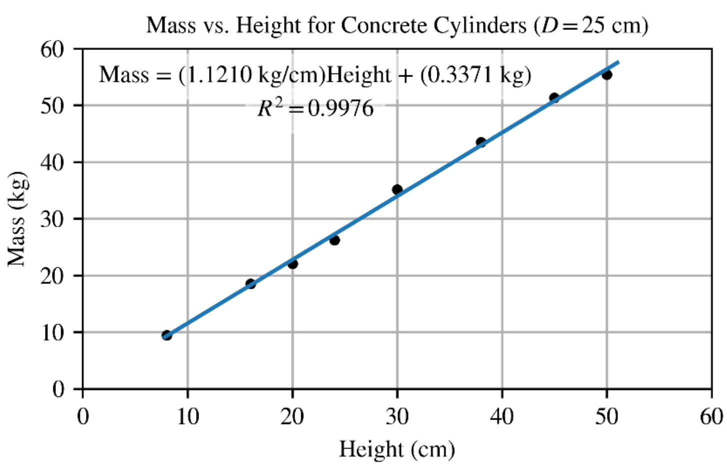


Figure 1: Example of graphing best practices. The data represent a hypothetical experiment where cylinders of the same diameter (25 cm) are made and their masses are measured.

1. **Title.** The title describes the nature of the data being graphed. The dependent variable is mentioned before the independent variable (the y-axis variable before the x-axis variable, or y vs. x).

   The title should *not* merely restate what's on the axis labels. A title like "velocity vs. time" would be inadequate: it doesn't describe the experiment any more than axis labels would. A more complete title would be "Average velocity of fall vs. time of fall." In Figure 1, the title is still short, but it is descriptive of the experiment.

2. **Proper scale.** The plotted points should use more than half of the available space along each axis. If Excel does not do this automatically, the axis bounds should be reset manually.

   In general, it is good practice to include the point (0, 0) on the graph. Do so unless it would contradict the previous paragraph or you have some other good reason not to.

3. **Axis labels.** Both axes should be labeled with **both** the quantity being measured and the units in which that quantity is measured. The quantity can be labeled *either* with the variable symbol *or* the English word, but *not* both. In Figure 1, the axes are labeled with English words, but the labels could have been "H (cm)" and "M (kg)" instead.

4. **Trend line.** Any graph of linear data should include a linear trend line. In the next section, we discuss how to deal with non-linear data.

5. **Equation of the line.** Any graph of linear data should include the equation of the trend line in the form $y = mx + b$, where y and x are replaced with the dependent and independent variables actually being plotted, and the slope m and intercept b have their numerical values **and units**.

   In Figure 1, the equation is Mass = (1.121 kg/cm)Height + (0.3371 kg). The English words, rather than variable symbols, are used to avoid ambiguity with unit abbreviations. If you want to use variable symbols instead, you *must* follow the scientific convention that variable symbols are italic and unit abbreviations are not, in which case you would have $M = (1.121\ \mathrm{kg/cm})H + (0.3371\ \mathrm{kg})$.

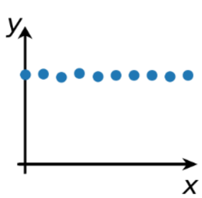
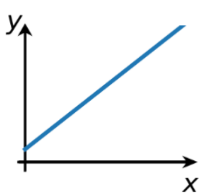
   The meaning of the slope of the graph above is that increasing the height of the cylinders by 1 cm increases the mass by 1.121 kg. The meaning of the intercept is that as the height of the cylinders goes to 0 cm, the mass would go to 0.3371 kg. This does not really make sense (we'd expect it to go to zero), but it will when we talk about uncertainty in the slope and intercept in Section 2.7.

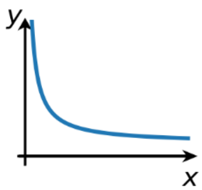
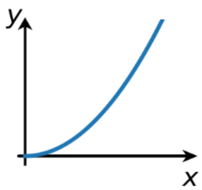
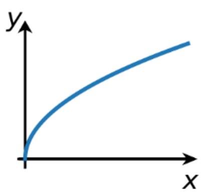
6. **R^2.** The square of the linear correlation coefficient is a statistical tool which measures how well the data points fit to the trend line. A value of 0 would indicate no linear correlation between the points, and a value of 1 would signify perfect linear correlation. The R^2 value is optional, but it may be useful when deciding whether the data needs to be linearized.

1.2. Linearization

Straight-line relationships are the easiest to understand. Often, if the relationship between the variables is *not* linear, we will change what we're graphing so that it *is* a straight line. There are many reasons to do this, though perhaps the most important is that it's (usually) easy to tell by looking whether some relationship is linear. If it is curved, it's often difficult to tell whether, for example, $y(x) \propto x^2$ or $y(x) \propto x^3$ just by looking.

This process of re-graphing different quantities to make the resulting graph linear is called **linearization**. Here is how to do it for some common graph shapes:

Graph Shape	Written Relationship	Modification Required to Linearize Graph	Algebraic Representation
	As x increases, y remains the same. There is no relationship between the variables.	Do not linearize the graph. Determine the average and standard deviation.	
	As x increases, y increases proportionally y is directly proportional to x.	None	$y = mx + b$

Graph Shape	Written Relationship	Modification Required to Linearize Graph	Algebraic Representation
	As x increases, y decreases. y is inversely proportional to x.	Graph y vs. $1/x$.	$y = m\left(\frac{1}{x}\right) + b$
	y is proportional to the square of x.	Graph y vs. x^2.	$y = mx^2 + b$
	y is proportional to the square root of x, or the square of y is proportional to x.	Graph y^2 vs. x.	$y^2 = mx + b$
		Graph y vs. $\sqrt{x}$.	$y = m\sqrt{x} + b$

2. Introduction to Uncertainty

2.1. Estimating Values and Uncertainties

Whenever we measure a quantity in lab, it is important to know not only the *value* of the quantity, but also *how sure* we are that the value is correct.

Every time we measure a quantity, there will be some *uncertainty* in the measurement. This uncertainty can come from any (and often more than one) of the following:

- The resolution of the instrument;
- Difficulty in precisely aligning instruments;
- Human limitations, like reaction time; and
- Inherent "fuzziness" of the quantity being measured.

Typically, to determine the uncertainty, we make an *estimate* based on what we know about our measurement instruments.

2.2. Measurements Relative to a Scale

Many of our measurements consist of comparing an object to a fixed scale. In PHYS 250 and 251, the most common examples are

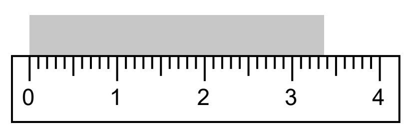


Figure 2: Measurements with a ruler.

- Rulers. We determine the length of something by comparing it to the markings on the ruler (see Figure 2).
- Triple-beam balances. We determine the mass of something by figuring out where to put the sliding masses to balance the object on the tray, and we read the

mass by looking at the locations of the sliding masses relative to the scales (basically rulers) printed on the beams.

The best estimate of the length of the box in Figure 2 is familiar: just read the scale to find $L_{\text{best}} = 3.35$ cm. But what is the uncertainty? Normally, we use the following rule.

Rule of thumb: When reading values from a scale, the uncertainty is half the size of the smallest interval marked on the scale.

For the ruler, the smallest marking is 0.1 cm, so the uncertainty is $\delta L = 0.05$ cm. The δ is a lowercase Greek letter delta. This notation is fairly standard: a δ before a variable means "the uncertainty in that variable."

The standard method for reporting a number with an uncertainty is as follows.

Standard notation for reporting a measured quantity x: Write the best estimate value plus or minus the uncertainty:

$$x = x_{\text{best}} \pm \delta x$$

The length of the object in Figure 2 is then $L = L_{\text{best}} \pm \delta L = (3.35 \pm 0.05)$ cm.

Let's make some comments on the meaning of this.

- We think the length of the object is $L_{\text{best}} = 3.35$ cm.
- However, because the ruler is only marked in 0.1 cm increments, we can only say with certainty that the length of the object is between $L_{\text{best}} - \delta L = 3.35 \text{ cm} - 0.05 \text{ cm} = 3.30$ cm and $L_{\text{best}} + \delta L = 3.35 \text{ cm} + 0.05 \text{ cm} = 3.40$ cm.

2.3. Significant Figures and Uncertainty

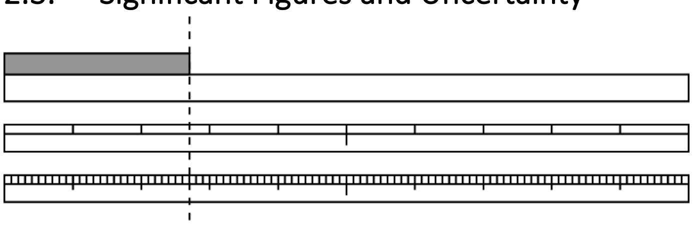


Figure 3: Measuring an object with three scales.

We'll reexamine the rule of thumb above to see when it applies, as well as how significant figures and uncertainties are related. To do this, we'll use Figure 3, which shows an object being measured with three differently-calibrated meter sticks.

The top meter stick has no markings except the left and right ends, which are 1 m apart. Then the rule of thumb is not reasonable—most people can accurately estimate the length more precisely than to the nearest 0.5 m. A more reasonable estimate of the uncertainty would be one *tenth* of the smallest marking, or 0.1 m. In the top drawing, then, we might say that the length of the object is (0.3 ± 0.1) m.

The middle meter stick is marked to the nearest 0.1 m, and the added markings allow us to make more precise estimates of the length. One tenth of the smallest intervals, 0.01 m, is again reasonable, giving a length of (0.27 ± 0.01) m.

Finally, we consider the bottom meter stick, marked to the nearest 0.01 m. These markings are too close to confidently estimate to the nearest tenth of that, and the rule of thumb comes into play: we choose half the smallest interval, or 0.005 m. We take the length to be (0.270 ± 0.005) m.

Looking at the last sentence of each of the three previous paragraphs reveals a pattern. As we increase the precision of our meter stick, the uncertainty decreases. Each time it gains a decimal place, the number of significant figures in the measured value increases by one.

Uncertainty and significant figures:

1. Experimental uncertainties always have one significant figure.
2. The best-estimate value of the measured quantity has the same number of decimal places as the uncertainty.
3. As a result, the number of significant figures in the best-estimate value is directly related to the experimental uncertainty and, therefore, to the precision of the measurement device. That is, **the number of significant figures provides information about the precision of the measurement.**

We will discuss significant figures further later (see Section 3).

2.4. Relative Uncertainty

In deciding whether a measurement is sufficiently precise, the *absolute uncertainty* (which we've so far just called the "uncertainty") is not very important. For example, if you are drawing lines to mark a football field, it doesn't really matter if the lengths are off by an inch. If you are measuring the thickness of your thumb and you're off by an inch, your measurement is completely wrong!

We quantify this using the concept of *relative uncertainty*, sometimes called *percent uncertainty*. Here is the definition:

Relative uncertainty in a quantity x:

$$\begin{aligned}\text{relative uncertainty} &= \frac{\text{absolute uncertainty}}{\text{measured value}} \times 100\% \\ &= \frac{\delta x}{x_{\text{best}}} \times 100\%\end{aligned}$$

Returning to Figure 2, the relative uncertainty in the length of the object from before is then

$$\frac{\delta L}{L} \times 100\% = \frac{0.05\text{ cm}}{3.35\text{ cm}} \times 100\% = 1.5\%.$$

Suppose that the width of that bar is $W = (0.45 \pm 0.05)$ cm. The relative uncertainty in the width is

$$\frac{\delta W}{W} \times 100\% = \frac{0.05\text{ cm}}{0.45\text{ cm}} \times 100\% = 11\%.$$

As a rule of thumb, **relative uncertainties should be smaller than 5**%. Our length measurement, then, is quite good, but the width measurement is not. Ideally, in such a situation, we would try to find a more precise measurement device (in this case, calipers would be good; see Section 5.2) or an alternate method of measurement, but in introductory labs, that is not always possible.

2.5. Uncertainties in Calculated Values

Often, what we want to know cannot be measured *directly*. We instead have to *calculate* it from things that we *can* measure. How do we find the uncertainty in the calculated value? We will explore this using two examples involving our bar: perimeter and area.

The best estimate for the perimeter of the bar is given by the usual formula:

$$P_{\text{best}} = 2L_{\text{best}} + 2W_{\text{best}} = 2(3.35\text{ cm}) + 2(0.45\text{ cm}) = 7.6\text{ cm}.$$

To find the uncertainty, we have to note that there are two things going on in the formula for P: multiplication by a precise constant, then addition. The rules for both are straightforward.

Uncertainties with multiplication by a constant: If a quantity Q is calculated by multiplying or dividing another quantity by a precise constant K,

$$Q = Kx,$$

the absolute uncertainty is multiplied by the same constant:

$$\delta Q = K\delta x.$$

The perimeter depends on $2L_{\text{best}}$ and $2W_{\text{best}}$. In both cases, the precise constant is $K = 2$, so

$$\delta(2L) = 2\delta L = 2(0.05\text{ cm}) = 0.1\text{ cm} \qquad \text{and} \qquad \delta(2W) = 2\delta W = 2(0.05\text{ cm}) = 0.1\text{ cm}.$$

We combine the two terms using the rule for addition and subtraction.

Uncertainties with addition and subtraction: If a quantity Q is calculated by adding or subtracting other quantities,

$$Q = (a + b + \cdots) - (x + y + \cdots),$$

the absolute uncertainties add:

$$\delta Q = (\delta a + \delta b + \cdots) + (\delta x + \delta y + \cdots).$$

Note that it does not matter whether you're adding or subtracting. The uncertainties *add* either way.

Since $P_{best} = (2L_{\text{best}}) + (2W_{\text{best}})$, the overall uncertainty is

$$\delta P = \delta(2L) + \delta(2W) = (0.1\text{ cm}) + (0.1\text{ cm}) = 0.2\text{ cm}.$$

Then, in standard form, the perimeter can be written

$$P = (7.6 \pm 0.2)\text{ cm}.$$

Now suppose we want to find the area. The best value is

$$A_{\text{best}} = L_{\text{best}}W_{\text{best}} = (3.35\text{ cm})(0.45\text{ cm}) = 1.5\text{ cm}^2.$$

The uncertainty can be estimated using the following rule.

Uncertainties with multiplication and division: If a quantity Q is calculated by multiplying or dividing other quantities,

$$Q = \frac{a \times b \times \cdots}{x \times y \times \cdots},$$

the *relative* uncertainties add:

$$\frac{\delta Q}{Q_{\text{best}}} = \frac{\delta a}{a_{\text{best}}} + \frac{\delta b}{b_{\text{best}}} + \cdots + \frac{\delta x}{x_{\text{best}}} + \frac{\delta y}{y_{\text{best}}} + \cdots$$

Applying this rule to the area, we have

$$\frac{\delta A}{A_{\text{best}}} = \frac{\delta L}{L_{\text{best}}} + \frac{\delta W}{W_{\text{best}}} = 0.015 + 0.11 = 0.12.$$

Note that we have canceled out the 100% in each term. The relative uncertainty in the area is then $0.12 \times 100\% = 12\%$. The absolute uncertainty is

$$\delta A = \frac{\delta A}{A_{\text{best}}} A_{\text{best}} = (0.12)\left(1.5\ \text{cm}^2\right) = 0.19\ \text{cm}^2.$$

We can then write the area in the standard form as

$$A = (1.5 \pm 0.2)\ \text{cm}^2.$$

2.6. Uncertainties in Repeated Measurements

When an individual measurement is repeated many times, it is natural to expect to get the same result every time. However, due to random, uncontrollable factors, this is often not the case. We usually will take as our best estimate the *mean* or *average*:

Best estimate in repeated measurements—mean or average: If a quantity x is measured N times, with the results being $x_1, x_2, \dots, x_N$, the best estimate of the value of x will usually be the average $\bar{x}$:

$$x_{\text{best}} = \bar{x} = \frac{x_1 + x_2 + \cdots + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i.$$

The uncertainty is then the *standard deviation*.

Uncertainty in repeated measurements—standard deviation: If a quantity x is measured N times, with the results being $x_1, x_2, \dots, x_N$, with an average value of $x_{\text{best}} = \bar{x}$, the uncertainty in the value of x will usually be the standard deviation of x, σ_x, given by

$$\delta x = \sigma_x = \sqrt{\frac{\left(x_1 - \bar{x}\right)^2 + \left(x_2 - \bar{x}\right)^2 + \cdots + \left(x_N - \bar{x}\right)^2}{N-1}} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2}.$$

For example, if you use a stopwatch to measure the time a ball takes to fall 3 meters, and you do this five times, you may get values like those in the table. Then, the best estimate of the time is the average:

Trial	Time (s)
1	0.61
2	0.63
3	0.60
4	0.59
5	0.64

$$t_{\text{best}} = \bar{t} = \frac{(0.61\ \text{s}) + (0.63\ \text{s}) + (0.60\ \text{s}) + (0.59\ \text{s}) + (0.64\ \text{s})}{5}$$
$$= 0.614\ \text{s}$$

When reporting the final result, we will need to take significant figures into account, but we want to keep extra digits for the next calculation.

To find the uncertainty, we calculate the standard deviation:

$$\delta t = \sigma_t = \sqrt{\frac{(0.61 - 0.614)^2 + (0.63 - 0.614)^2 + (0.60 - 0.614)^2 + (0.59 - 0.614)^2 + (0.64 - 0.614)^2}{5-1}}\ \text{s}$$
$$= 0.02\ \text{s}$$

The value of the fall time is then $t = (0.61 \pm 0.02)$ s.

In practice, mean and standard deviation will be calculated in Excel, rather than by hand, using the functions `=AVERAGE(<CELLS>)` and `=STDEV.S(<CELLS>)`.

2.7. Uncertainty in Fit Parameters

Excel's Regression Analysis tool allows you to find the uncertainty in the slope and intercept of fitted data. It's not enabled by default, but it is not difficult.

Enabling Regression Analysis

The following steps need to be followed in Excel only once (per user).

1. Go to **File** → **Options**.
2. Click on **Add-ins**.
3. Under "Inactive Application Add-ins," click on **Analysis ToolPak**.
4. Near the bottom of the dialog, click the **Go** button next to "Manage: Excel Add-ins."
5. In the dialog that opens, check **Analysis ToolPak**, then click **OK**.
6. If you are returned to the Options dialog, click **OK**.

Doing Regression Analysis

To do regression analysis, follow these steps:

1. Go to the **Data** ribbon.
2. In the "Analysis" block toward the right, click on **Data Analysis**.
3. Select **Regression**, and click **OK**.
4. Select the appropriate ranges of cells for "Input Y Range" and "Input X Range." Then click **OK**.
5. A new sheet will be created with a lot of statistical information. The relevant part is in the bottom-left. For the data in Figure 1 (back in Section 1.1), the result is this:

	Coefficients	*Standard Error*
Intercept	0.337128	0.712021
X Variable 1	1.120978	0.022287

The *Intercept* row gives information about the y-intercept, and the *X Variable 1* row tells you about the slope. The *Coefficients* column gives the best-estimate values of the intercept and slope, and the *Standard Error* column provides the uncertainties in the intercept and slope.

For these data, the slope is then (1.12 ± 0.02) kg/cm. The intercept is (0.3 ± 0.7) kg, meaning that any intercept between $(0.3 - 0.7)$ kg $= -0.4$ kg and $(0.3 + 0.7)$ kg $= 1$ kg is consistent with the experiment. This range includes 0 kg, so the experimental data are consistent with the expectation that a cylinder with a diameter of 0 cm would have no mass.

3. Significant Figures

3.1. Significant Figures in Measured Quantities

To summarize Section 2.3, the number if significant digits in a measured quantity is determined by the uncertainty as follows:

1. Determine the best estimate and the uncertainty. Make sure that they are in the same units.
2. Keep only one *digit* in the uncertainty.
3. Keep the same number of *decimal places* in the best estimate as there are in the uncertainty.

For example, suppose measurement yields $x_{\text{best}} = 3.50192$ m and $\delta x = 0.0124$ m. We can keep only one digit in the uncertainty, so we round it to $\delta x = 0.01$ m. This goes to two decimal places, so we keep two decimal places in the best estimate, rounding to $x_{\text{best}} = 3.50$ m.

3.2. Zeros and Significant Figures

The rules for interpreting zeros when considering significant digits are

1. All non-zero digits in a measurement **are** considered significant.
2. Zeros **are** significant when they are bounded by non-zero digits.
3. If a decimal point is present, all zeros following non-zero digits **are** significant (e.g. a measurement of 30.00 kg has four significant digits).
4. If a decimal point is not present, all zeros following the last non-zero digit are **not** significant—they are placeholders only (e.g. a measurement of 160 N has two significant figures).
5. Zeroes preceding the first non-zero digit are **not** significant—they are placeholders (e.g. a measurement of 0.00610 m has three significant figures).

3.3. Significant Digits When Multiplying and Dividing

When multiplying numbers, the result has the same number of significant digits as the number with the *fewest* significant digits. Here are some examples:

- $3.142 \times 2.91 = 9.14322$. The second number, 2.91, has three significant figures, while 3.142 has four. The result should then have three digits, and we round to 9.14.
- $0.3 \times 19.1 = 5.73$. Here the first number has only one significant digit, so the answer rounds to 6.

3.4. Significant Digits When Adding and Subtracting

When numbers are added or subtracted, the result is rounded so that it has the same number of *decimal places* as the number with the fewest decimal places. Here is an example:

$$\begin{array}{r} 3.0264 \\ 21.04 \\ \underline{+\ \ 7.99219} \\ 32.05859 \end{array}$$

Since 21.04 has only two decimal places, the result must be rounded to the second decimal place, giving 32.06.

3.5. Rules for Rounding

Note: Only round the final answer. Do not round intermediate steps.

In grade school, most people are taught that 0.5 rounds up to 1, while $0.4\overline{9}$ rounds down. In science, rounding like this introduces systematic error, and we instead follow these rules:

1. If the leftmost digit to be removed is greater than 5, round up (e.g. 4.66 becomes 4.7).
2. If the leftmost digit to be removed is less than 5, round down (e.g. 4.64 becomes 4.6).
3. If the leftmost digit to be removed equals 5:
   a. If the preceding digit is odd, round up (e.g. 4.75 becomes 4.8)
   b. If the preceding digit is even, round down (e.g. 4.65 becomes 4.6)

4. Putting Things Together: An Example

Consider a slight variation on the example of Figure 1. Suppose instead of varying the height of the cylinder, we vary the diameter.

4.1. Making a Graph

We follow best practices from Section 1.1 to graph the data.

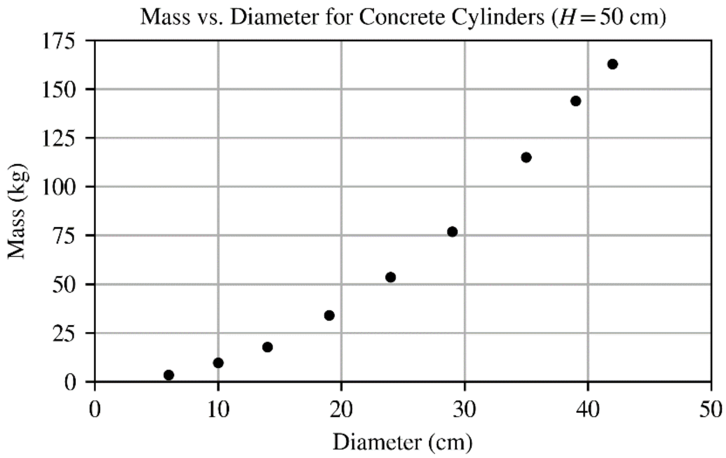


Figure 4: Measured mass vs. varied diameter for cylinders of fixed height. The data are not linear, so we do not include trendline, equation, or R^2. Axes are still labeled with both quantity name and units, and the data take up most of the range, both horizontally and vertically.

4.2. Linearization

The scatter plot does not look linear. It looks roughly quadratic. We can refer to the table in Section Linearization1.2 and note the shape is similar to the fourth row. Therefore, we leave the vertical axis unchanged and square the diameter in the horizontal axis.

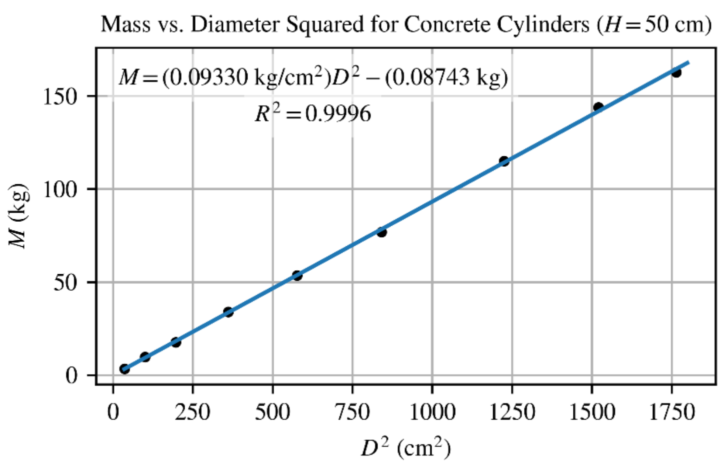


Figure 5: Linearized data from Figure 4. Since we are doing math on the quantities, we have switched to variable symbols. Note that the variables are italic while the units are upright. We now include a trendline and an equation.

The data are now very linear (the correlation coefficient is very close to 1).

4.3. Uncertainty in Slope and Intercept

Regression analysis (Section 2.7) gives the following results:

	Coefficients	*Standard Error*
Intercept	-0.08743	0.646363
X Variable 1	0.093298	0.000679

From the Intercept row, we get

$$(\text{intercept}) = (-0.1 \pm 0.6)\ \text{kg}.$$

From the X Variable 1 row, we get (slope) = (0.0933 ± 0.0007) kg/cm^2. We can convert this to normal SI units using 100 cm = 1 m:

$$\text{(slope)} = (933 \pm 7)\ \text{kg/m}^2.$$

Note that we have followed the rules for significant figures outlined in Sections 2.3 and 3.1.

4.4. Interpreting the Equation and Comparing to Theory

You may recall that the volume of a cylinder of height H, radius R, and diameter $D = 2R$ is

$$V = \pi H R^2 = \pi H (D/2)^2 = \pi H D^2/4.$$

The definition of density is $\rho = M/V$, where M is mass. Rearrange this to get $M = \rho V$, and substitute the expression for volume:

$$M = \frac{\pi}{4}\rho H D^2 + 0.$$

Compare this to the standard form of a line, $y = mx + b$, which in our graph is

$$M = \text{(slope)}D^2 + \text{(intercept)}.$$

We see that the intercept should theoretically be zero, which agrees with the fit. The slope is whatever is multiplied by the independent variable (D^2), so

$$\text{(slope)} = \frac{\pi}{4}\rho H.$$

Therefore, we can find the density of the cylinders by rearranging:

$$\rho = \frac{4}{\pi H}\text{(slope)} = \frac{4}{\pi(0.50\ \text{m})}\left(\frac{933\ \text{kg}}{\text{m}^2}\right) = 2375.814\ \text{kg/m}^3.$$

We can find the uncertainty in the density using the methods from Section 2.5. Suppose the uncertainty in H is 0.1 cm. Since the slope and H combine by division and then are multiplied by a precise constant $(4/\pi)$, we can use the rule for multiplication and division (add the relative errors), and then multiply the result by the same constant:

$$\frac{\delta\rho}{\rho} = \frac{4}{\pi}\left(\frac{\delta\text{(slope)}}{\text{slope}} + \frac{\delta H}{H}\right) = \frac{4}{\pi}\left(\frac{7}{933} + \frac{0.1}{50}\right) = 0.011813.$$

The absolute uncertainty is then $\delta\rho = (\delta\rho/\rho)\rho = (0.011813)\left(2375.814\ \text{kg/m}^3\right) = 28.065\ \text{kg/m}^3$. Rounding to the correct number of significant figures, we have

$$\rho = (2380 \pm 30)\ \text{kg/m}^3.$$

Note that since there is no decimal point, the zeros are not significant—they are merely placeholders—as discussed in Section 3.2.

5. Some Measurement Equipment

5.1. Most Basic Lab Equipment

Rulers, Meter Sticks, and Protractors

The use of rulers, meter sticks, and protractors is straightforward and familiar, and in all three cases, the rule of thumb—the uncertainty in measurements is half the smallest marked interval—usually applies.

The exception is when it is difficult to align the measuring device with the object being measured. When that is the case, the uncertainty is larger.

Triple-Beam Balances

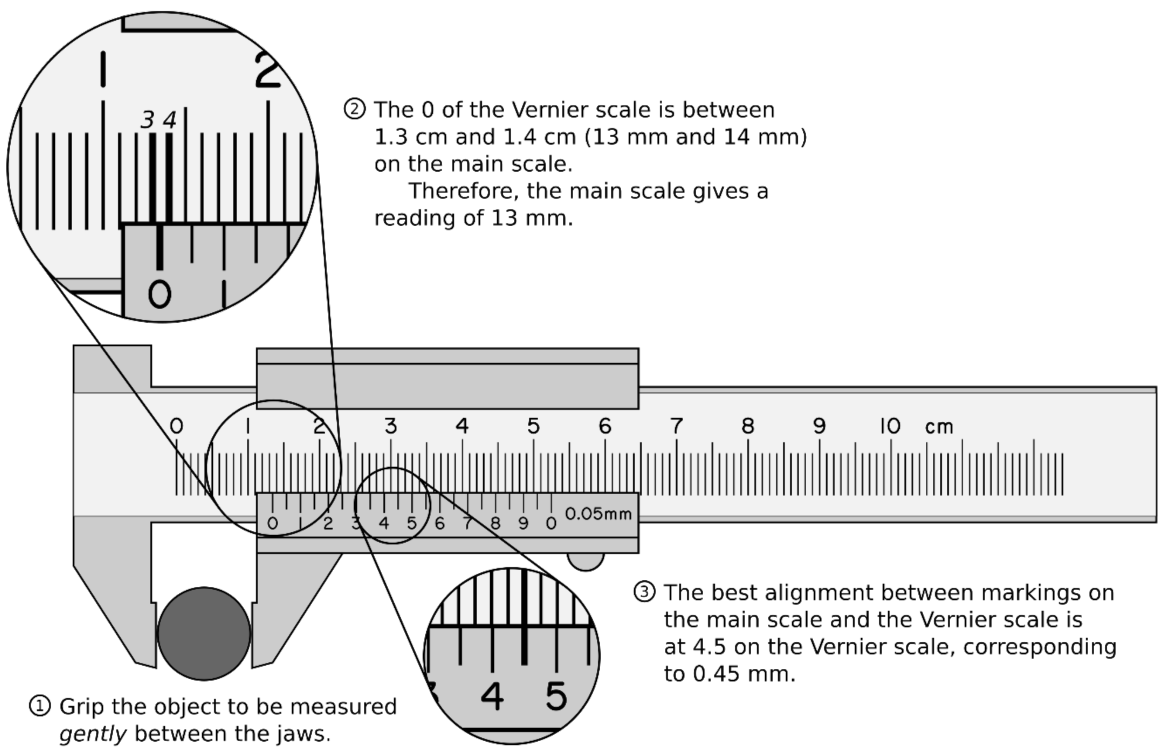
The triple-beam balance is an extremely precise measuring tool. The smallest marking is 0.1 g, Alignment is not an issue with a balance, so the exceptions to the rule of thumb are rare. The uncertainty is 0.05 g, or 0.00005 kg.

Stopwatches

Most stopwatches report times to the nearest 0.01 s. The rule of thumb would suggest that the uncertainty is 0.005 s, but this is wrong. Human reaction time is significantly longer than 0.01 s, so the uncertainty in times measured using a stopwatch are much larger. Reaction time varies from person to person, but a typical value is about 0.2 s, so this is a reasonable value to use as the uncertainty.

5.2. Vernier Calipers

Vernier calipers are tools for measuring lengths very precisely. They consist of two scales: a fixed scale (called the *main scale*) marked in centimeters and a sliding scale, called the *Vernier scale*. The basic operation is shown here:



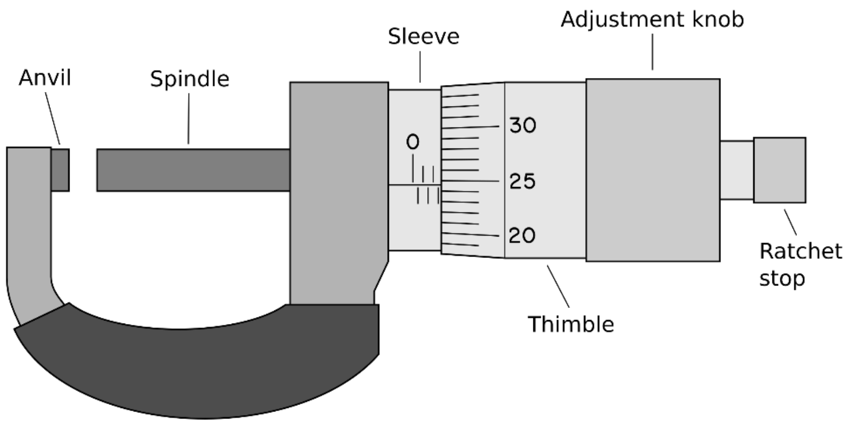
1. Grip the object to be measured in the jaws, or, to measure the inner diameter of a hole, spread the jaws until the upper prongs (not shown in the figure) touch the edges of the whole.
2. Look for where the 0 on the Vernier scale is on the main scale. The main-scale value directly to the left is the number of whole millimeters. In the figure, the big inset shows this to be 13 mm.
3. Look at the markings (ignoring the numbers for now) on the two scales, paying attention to where they line up neatly. There are two possibilities:
   a. One line on the Vernier scale will line up exactly with a line on the main scale. The value on the Vernier scale where this happens is the number of tenths of a millimeter. In the figure, the smaller inset shows this to be at 4.5, so the reading is (4.5)(0.1 mm) = 0.45 mm.

    b. Two Vernier lines will appear to be equally well aligned and better aligned than any other markings. If that happens, take the average of the two values.
4. Finally, add up the number of whole millimeters from Step 2 and the decimal part from Step 3 to get the length. In the figure, it's 13 mm + 0.45 mm = 13.45 mm.

The uncertainty in measurements with Vernier calipers is usually marked on the calipers somewhere. For most, it's on the right edge of the Vernier scale (in the figure, it's 0.05 mm; some mark it as 1/20 mm, which has the same value of 0.05 mm).

5.3. Micrometers

For more precision than calipers provide, you can use a micrometer. The figure shows its main parts.

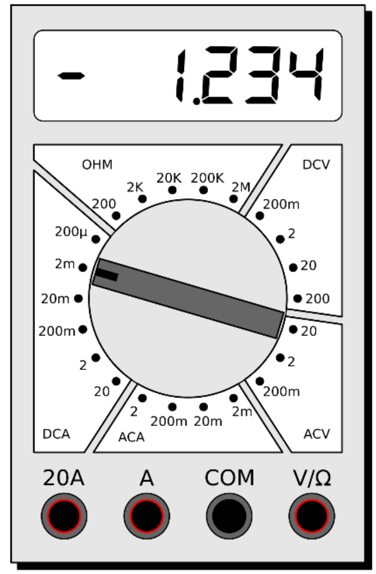


1. Place the object you want to measure between the anvil and the spindle.
2. Rotate the adjustment knob until the anvil and spindle nearly touch the object to measure.
3. Use the ratchet stop to tighten the rest of the way without over-tightening.
4. Look at the exposed lines on the sleeve.
    a. The upper marks give the number of whole millimeters (in the figure, it's 2 mm).
    b. If a lower mark is visible after the last upper mark, add 0.5 mm (in the figure, one is, so we have 2.5 mm).
5. Now look at the markings on the thimble. They are marked in units of 0.01 mm. The thimble reading is the number on the thimble that aligns with the central, horizontal line on the sleeve. (In the figure, the reading is 24.7, or 0.247 mm.)
6. Add the thimble reading from Step 5 to the sleeve reading from Step 4 to get the final size of the object (in the figure, we have 2.5 mm + 0.247 mm = 2.747 mm).

Since the smallest markings on the micrometer are 0.01 mm apart, the rule of thumb gives an uncertainty of 0.005 mm.

5.4. Multimeters

The basic measurement tool for circuits is the multimeter, so called because it combines a *voltmeter* to measure voltage, an *ohmmeter* to measure resistance, and an *ammeter* to measure current (some higher-end models can also measure capacitance and even inductance).



To use a multimeter, you have to choose three things:

1. which input ports to use on the multimeter,
2. where to connect the multimeter to the circuit, and
3. where to position the multimeter setting dial.

Half of point 1 is easy: **the negative wire always goes into the port labeled COM** (short for "common"). The positive wire goes into **V/Ω** for voltage or resistance measurements, **A** for measurements of currents less than 2 amps, or **20A** to measure currents greater than 2 amps and less than 20 amps.

Measuring Voltage

The multimeter should be connected **in parallel** with the part of the circuit you want to measure the voltage across.

The dial should be within the **DCV** section for a dc voltage or within **ACV** for an ac voltage.

The basic units for voltage are volts (symbol V). A positive value means that whatever is attached to port V is at higher potential than port COM. A negative value means that COM is at higher potential.

Measuring Resistance

To measure a resistance, first **take the resistor or resistors you want to measure out of the rest of the circuit**. Then connect the multimeter **in parallel** with the resistor (or combination of resistors).

The dial should be within the **OHM** section.

The basic units for resistance are ohms (symbol Ω).

Measuring Current

To measure the current through some component, remove a bit of wire (or a snap-circuit piece) which is **in series** with that component. Then attach the multimeter wires *in place* of that bit of wire.

Turn the multimeter dial so that it is in the **DCA** section for a dc current or the **ACA** section for an ac current.

The basic units for current are amperes, or amps (symbol A). A positive value means that current flows into port A (or 20A, whichever is connected) and out of port COM. A negative value means that current flows in the opposite direction.

Choosing a Range/Precision

Within each section, there are several options, each of which indicates the maximum value the meter can read if the dial is set to that option. The labels also tell you what SI prefix to apply to the units. Examples:

- If the dial is set to 200µ in the DCA section, the maximum current that the multimeter can read is 200 µA, and the number on the display is in microamps.
- If the dial is set to 200K in the OHM section, the maximum resistance you can measure is 200 kΩ, and the reading is in kiloohms.

- In the figure, the dial is set to 2m DCA, so the current is 1.234 mA, flowing into COM and out of A.

For maximum precision, turn the dial to the smallest setting larger than the value to be measured (e.g. for a 19 kΩ resistor, use the 20K OHM setting; for a 21 kΩ resistor, you have to use the 200K OHM setting).

If the display shows nothing but a 1, this means that the value you are trying to measure is too large for the current setting.

The uncertainty rule of thumb generally works for multimeters: the uncertainty is half the smallest value for the current setting. The estimated last digit (not shown on the screen) should be 0 if the reading is stable or 5 if the last digit bounces between two consecutive values. For example, the current measured in the figure is

- (-1.2340 ± 0.0005) mA if the value is not changing, or
- (-1.2345 ± 0.0005) mA if the reading flips back and forth between -1.234 and -1.235.